## Assignment 0 CS289: Algorithmic Machine Learning, Fall 2017 **NOT TO BE SUBMITTED**

You need background in **linear algebra**, **probability theory**, **and algorithms** (all at a typical undergraduate upper-division level) to make the class fun and interesting for you as well as for me. You can judge your background with this problem set (not to be submitted). If you have doubts about this please talk to me right away.

## Linear algebra basics

- 1. Define a subspace and dimension of a subspace.
- 2. Let  $S = \{x \in \mathbb{R}^d : \sum_{i=1}^d x_i = 0\}$ . What is the dimension of S?
- 3. Define orthonormal basis for a subspace S.
- 4. What is the projection of the vector  $v = (1, 0, ..., 0) \in \mathbb{R}^d$  onto the vector  $u = (1, 1, ..., 1) \in \mathbb{R}^d$ ? What is the length of the projection?
- 5. Define rank of a matrix. Let  $A \in \mathbb{R}^{m \times r}$ ,  $B \in \mathbb{R}^{r \times n}$  and let C = AB. Prove that rank of C is at most r.

## **Probability basics**

- 1. Define a random variable, expectation of a random variable, and variance of a random variable.
- 2. Suppose you toss a coin whose probability of heads is 1/10. What is the expected number of tosses until you see a heads?
- 3. When are two random variables X, Y independent?
- 4. Suppose you a roll a fair die *n* times. Let X be the number of 6's in the throws and  $\mathcal{E}$  be the event that all throws are even numbers. What is expectation of X? What is expectation of X conditioned on  $\mathcal{E}$ ?
- 5. Suppose you toss a coin whose probability of heads is 1/10 n times. Let X denote the number of heads. What is variance of X?